4.3 Properties of Logarithms MATH 1610 THOMPSON

Product Rule: $\log_b xy = \log_b x + \log_b y$

Quotient Rule: $\log_b \frac{x}{z} = \log_b x - \log_b z$

Power Rule: $a \log_b x = \log_b x^a$

1.) $\log_1 1 = 0$

2.) $\log_a a^r = r$

3.) $\log_a (MN) = \log_a M + \log_a N$

4.) $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

5.) $\log_a M^r = r\log_a M$

6) $\log_8 8^{87} = 87$

7.) $\ln e^6 = 6$

8.) $\log_{56} 8 + \log_6 7 = \log_{56} (8)(7) = \log_{56} 56 = x$

$56^x = 56$ $x=1$

9.) $\log_4 24 - \log_4 6$ $\log_4 \frac{24}{6} = \log_4 4 = x$ $4^x = 4$ $x = 1$

10) $3^{\log_3 5 - \log_3 8} = \frac{5}{8}$

11) $\log_3(81x) = \log_3 81 + \log_3 x = 4 + \log_3 x$

12) $\log_5 Z^4 = 4\log_5 Z$

13) $\ln(ex) = \ln e + \ln x$

$1 + \ln x$

14) $\ln[\frac{x}{e^{2x}}]$

$\ln x - 2x \ln e$

$\ln x - 2x$

15) $\log_b(u^6 v^5)$ *u>0, v>0 just means positive*

$6\log_b u + 5\log_b v$

16) $\ln\left(x^4\sqrt{3-x}\right)$

$4\ln x + \frac{1}{2}\ln(3-x)$

17) $\log_5 \left(\frac{x^{15}}{x-4}\right)$

$15\log_5 x - \log_5 (x-4)$

18) Write the expression as a sum and/or difference of logarithms.
factors.

$$\log\left[\frac{x(x+7)}{(x+5)^5}\right],\ x>0$$

$\log x + \log(x+7) - 5\log(x+5)$

19) Write the expression as a sum and/or difference of logarithms. Express powers as factors.
factor (x-4)(x+3) multiply 5 and ½

$$\ln\left[\frac{x^2-x-12}{(x+2)^5}\right]^{1/2},\ x>4$$

$\frac{1}{2}\ln(x-4) + \frac{1}{2}\ln(x+3) - \frac{5}{2}\ln(x+2)$

20) Write the expression as a single logarithm.

$$6\log_3 u + 9\log_3 v$$

$\log_3 u^6 v^9$

21) Write the expression as a single logarithm.

$$\log_2\sqrt{x} - \log_2 x^8$$

$\log_2 \frac{\sqrt{x}}{x^8} = \log_2 \frac{x^{1/2}}{x^8}$ ← subtract exponents

$\frac{1}{2} - 8 = -\frac{15}{2}\log_2 x$

22) Write the expression as a single logarithm.

$$\log_4(x^2-9) - 5\log_4(x+3)$$

$\log_4 \frac{(x+3)(x-3)}{(x+3)^5} = \log_4 \frac{(x-3)}{(x+3)^4}$ ← subtract 1 from bottom exponent

23) Write the expression as a single logarithm.

$$4\log_a(9x^9) - \frac{1}{8}\log_a(6x+11)$$

$9^4\ x^{9\cdot 4}$

$\log_a \frac{(9x^9)^4}{(6x+11)^{1/8}} = \log_a \frac{6561x^{36}}{\sqrt[8]{6x+11}}$

24.) Write the expression as a single logarithm.

$3\log_5(x+8) - \log_5(x-12) - \log_5(x-3)$

$= log_5 \left[\frac{(x+8)^3}{(x-12)(x-3)}\right]$

25.) If $f(x) = \ln x$, $g(x) = e^{7x}$, and $h(x) = x^4$, find the following.

(a) $(f \circ g)(x)$. What is the domain of $f \circ g$? $\ln e^{7x} = 7x$ D: $(-\infty, \infty)$

(b) $(g \circ f)(x)$. What is the domain of $g \circ f$? $e^{7\ln x} = x^7$ D: $(0, \infty)$

(c) $(f \circ g)(3)$ $7(3) = 21$

(d) $(f \circ h)(x)$. What is the domain of $f \circ h$? $\ln x^4$ D: $(-\infty, 0) \cup (0, \infty)$

(e) $(f \circ h)(e)$ $\ln e^4 = 4$

$\ln x^4$ graph →

domain $x \neq 0$